

**Handout on  
DEMAND ELASTICITY AND TOTAL EXPENDITURE  
Econ 101 — Professor Quirnbach**

Let the demand be evaluated at two prices,  $P_A$  and  $P_B$ , where  $P_A > P_B$ . Suppose  $D_A$  is the quantity demanded at  $P_A$  and that  $D_B$  is the quantity demanded at  $P_B$ . Total expenditure is the product of price times quantity, here  $P_A \cdot D_A$  and  $P_B \cdot D_B$ .

What we want to show is that expenditure is higher at the higher price if and only if demand is inelastic. Using the arc method, demand being inelastic means

$$\eta_{D,P}^{Arc} = \frac{P_A + P_B}{D_A + D_B} \cdot \frac{D_A - D_B}{P_A - P_B} > -1 \quad (1)$$

Simple algebra then gives

$$(P_A + P_B) \cdot (D_A - D_B) > -(P_A - P_B) \cdot (D_A + D_B) \quad (2)$$

$$P_A D_A + P_B D_A - P_A D_B - P_B D_B > -P_A D_A - P_A D_B + P_B D_A + P_B D_B \quad (3)$$

$$P_A D_A - P_B D_B > -P_A D_A + P_B D_B \quad (4)$$

$$2P_A D_A > 2P_B D_B \quad (5)$$

$$P_A D_A > P_B D_B \quad (6)$$

To go from (1) to (2), multiply both sides of (1) by  $(D_A + D_B) \cdot (P_A - P_B)$ . Multiply out the expressions in (2) to get (3). Then subtract  $P_B D_A - P_A D_B$  from both sides to get (4). Then add  $P_A D_A + P_B D_B$  from both sides to get (5) and divide by 2 to get (6).

If you reverse the inequality in (1), all the rest reverse, too. Thus, expenditure decreases at the higher price if and only if demand is elastic. And, if (1) holds as an equality, again so do the rest. Thus, if demand is unit elastic ( $\eta_{D,P}^{Arc} = -1$ ), then expenditure is exactly the same at both prices.